

## WEIGHTED REGULARITY ESTIMATES IN ORLICZ SPACES FOR THE PARABOLIC SCHRÖDINGER OPERATORS

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*Abstract.* In this paper we study regularity estimates in weighted Orlicz spaces for the parabolic Schrödinger operator

$$P = \frac{\partial}{\partial t} - \Delta + V(x, t)$$

with non-negative potentials  $V(x, t)$  satisfying certain reverse Hölder class. As a corollary we obtain the classical  $L^p$ -type regularity estimates for such operator.

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