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HERMITE'S FORMULA FOR q–GAMMA FUNCTION

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Abstract. In this paper, we presented the Raabe's integral and Hermite's formula for q-gamma function $\Gamma_q(x)$, $0<q<1$. We deduced new proofs of the formulas $\frac{\Gamma_q'(x)}{\Gamma_q(x)}$ and q-Gauss's multiplication using the Hermite's formula of $\Gamma_q(x)$ and H. Jack's technique [11]. Also, we deduced new double inequality of $\Gamma_q(x)$.

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