

HARDY–HILBERT’S INEQUALITY AND POWER INEQUALITIES FOR BEREZIN NUMBERS OF OPERATORS

MUBARIZ T. GARAYEV, MEHMET GÜRDAL AND ARZU OKUDAN

Abstract. We give operator analogues of some classical inequalities, including Hardy and Hardy–Hilbert type inequalities for numbers. We apply these operator forms of such inequalities for proving some power inequalities for the so-called Berezin number of self-adjoint and positive operators acting on Reproducing Kernel Hilbert Spaces (RKHSs). More precisely, we prove that

$$(\text{ber}(f(A)))^2 \leq C \text{ber}\left((f(A))^2\right)$$

for some constants $C > 1$. We also use reproducing kernels technique to estimate $\text{dist}(A, \mathcal{U})$, where \mathcal{U} is the set of all unitary operators on a RKHS $\mathcal{H} = \mathcal{H}(\Omega)$ over some set Ω , for some operator A on $\mathcal{H}(\Omega)$.

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