

## GLOBAL TRUDINGER–MOSER INEQUALITY ON METRIC SPACES

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*Abstract.* We study Trudinger-Moser type inequalities on the entire metric measure spaces. Moreover, we give the necessary and sufficient conditions under which the Trudinger-Moser inequality holds.

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