

POPOVICIU TYPE INEQUALITIES VIA HERMITE'S POLYNOMIAL

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Abstract. We obtain useful identities via Hermite interpolation polynomial, by which the inequality of Popoviciu for convex functions is generalized for higher order convex functions. We investigate the bounds for the identities extracted by the generalization of the Popoviciu inequality using inequalities for the Čebyšev functional. Some results relating to the Grüss and Ostrowski type inequalities are constructed.

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