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BERNSTEIN–MARKOV TYPE INEQUALITIES AND OTHER INTERESTING ESTIMATES FOR POLYNOMIALS ON CIRCLE SECTORS

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Abstract. In this paper we study various polynomial inequalities for 2-homogeneous polynomials on the circular sector $\{re^{i\theta} : r \in [0,1], \theta \in [0, \frac{\pi}{2}]\}$. In particular, we obtain sharp Bernstein and Markov inequalities for such polynomials, we calculate the polarization constant of the space formed by those polynomials and, finally, we provide the unconditional basis constant of the canonical basis of that polynomial space.

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