

## IMPROVED JENSEN'S INEQUALITY

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*Abstract.* In this article we present refinements of Jensen's inequality and its reversal for convex functions, by adding as many refining terms as we wish. Then as a standard application, we present several refinements and reverses of well known mean inequalities.

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