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ON THE ORLICZ SYMMETRY OPERATOR

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Abstract. R. Schneider (1970) proved that if $K \in \mathbb{R}^n$ is a convex body, such that each shadow boundary of K with respect to parallel illumination halves the Euclidean surface area of K, then K is centrally symmetric. A generalization of the results of R. Schneider was given by G. Averkov, E. Makai and H. Martini (2009). In this paper, by introducing an Orlicz symmetry operator $\Delta_\phi : \mathscr{K}^n \to \mathscr{K}^n$, we show a new method to obtain the characterization of symmetry for convex bodies. As an application, we will show that there is a unique member of $\Delta_\phi\langle K\rangle$ characterized by having larger volume than that of any other member of $\Delta_\phi\langle K\rangle$, where $\Delta_\phi\langle K\rangle$ is the Orlicz symmetric equivalence class of K.

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