

## THE MULTI-PARAMETER HAUSDORFF OPERATORS ON $H^1$ AND $L^p$

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*Abstract.* In the present paper, we characterize the nonnegative functions  $\varphi$  for which the multi-parameter Hausdorff operator  $\mathcal{H}_\varphi$  generated by  $\varphi$  is bounded on either the multi-parameter Hardy space  $H^1(\mathbb{R} \times \cdots \times \mathbb{R})$  or  $L^p(\mathbb{R}^n)$ ,  $p \in [1, \infty]$ . The corresponding operator norms are also obtained. Our results improve some recent results in [4, 15, 16, 18] and give an answer to an open question posted by Liflyand [12].

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