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ON THE MIXED (ℓ_1, ℓ_2)–LITTLEWOOD INEQUALITIES AND INTERPOLATION

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Abstract. It is well-known that the optimal constant of the bilinear Bohnenblust–Hille inequality (i.e., Littlewood's $4/3$ inequality) is obtained by interpolating the bilinear mixed (ℓ_1, ℓ_2)-Littlewood inequalities. We remark that this cannot be extended to the 3-linear case and, in the opposite direction, we show that the asymptotic growth of the constants of the m-linear Bohnenblust–Hille inequality is the same of the constants of the mixed $\left(\ell_{\frac{2m+2}{m+2}}, \ell_2\right)$-Littlewood inequality. This means that, contrary to what the previous works seem to suggest, interpolation does not play a crucial role in the search of the exact asymptotic growth of the constants of the Bohnenblust–Hille inequality. In the final section we use mixed Littlewood type inequalities to obtain the optimal cotype constants of certain sequence spaces.

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