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INEQUALITIES FOR THE FUNDAMENTAL ROBIN EIGENVALUE FOR THE LAPLACIAN ON N–DIMENSIONAL RECTANGULAR PARALLELEPIPEDS

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Abstract. Amongst N-dimenstional rectangular parallelepipeds (boxes) of a given volume, that which has the smallest fundamental Robin eigenvalue for the Laplacian is the N-cube. We give an elementary proof of this isoperimetric inequality based on the well-known formulae for the eigenvalues. Also treated are various related inequalities which are amenable to investigation using the formulae for the eigenvalues.

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