

PHILOS' INEQUALITY ON TIME SCALES AND ITS APPLICATION IN THE OSCILLATION THEORY

BAŞAK KARPUZ

Abstract. In [Bull. Acad. Polon. Sci. Sér. Sci. Math. 29 (1981), no. 7-8, 367–370], Philos proved the following result: Let $f: [t_0, \infty)_{\mathbb{R}} \rightarrow \mathbb{R}$ be an n -times differentiable function such that $f^{(n)}(t) \leq 0$ ($\neq 0$) and $f(t) > 0$ for all $t \geq t_0$. If f is unbounded, then $f(t) \geq \frac{\lambda^{n-1}}{(n-1)!} f^{(n-1)}(t)$ for all sufficiently large t , where $\lambda \in (0, 1)_{\mathbb{R}}$. In this work, we first present time scales unification of this result. Then, by using it, we provide sufficient conditions for oscillation and asymptotic behaviour of solutions to higher-order neutral dynamic equations.

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