

SHARP REDHEFFER–TYPE AND BECKER–STARK–TYPE INEQUALITIES WITH AN APPLICATION

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Abstract. In this paper, we give sharp Redheffer-type and Becker-Stark-type inequalities for trigonometric functions. As an application of Redheffer-type inequality, we improve the well-known Yang Le inequality.

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REFERENCES

- [1] M. ABRAMOWITZ AND I. A. STEGUN (Eds.), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Dover, New York, 1965.
- [2] D. AHARONOV AND U. ELIAS, *Improved inequalities for trigonometric functions via Dirichlet and Zeta functions*, *Math. Inequal. Appl.* **16**, 3 (2013), 851–859.
- [3] G. D. ANDERSON, S.-L. QIU, M. K. VAMANAMURTHY AND M. VUORINEN, *Generalized elliptic integral and modular equations*, *Pacific J. Math.* **192** (2000), 1–37.
- [4] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, New York, 1997.
- [5] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Monotonicity of Some Functions in Calculus*, Available at <http://www.math.auckland.ac.nz/Research/Reports/Series/538.pdf>.
- [6] B. BANJAC, M. MAKRAGIĆ AND B. MALEŠEVIĆ, *Some notes on a method for proving inequalities by computer*, *Results. Math.* **69**, 1 (2016), 161–176.
- [7] M. BECKER AND E. L. STRAK, *On a hierarchy of quolynomial inequalities for $\tan x$* , *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. No. 602–633 (1978)*, 133–138.
- [8] B. A. BHAYO AND J. SÁNDOR, *On Jordan's, Redheffer's and Wilker's inequality*, *Math. Ineq. Appl.* **19**, 3 (2016) 823–839.
- [9] C.-P. CHEN AND W.-S. CHEUNG, *Sharp Cusa and Becker–Stark inequalities*, *J. Inequal. Appl.* **2011** (2011) 136, <http://www.journalofinequalitiesandapplications.com/content/2011/1/136>.
- [10] C.-P. CHEN AND R. B. PARIS, *Series representations of the remainders in the expansions for certain trigonometric functions and some related inequalities, I*, *Math. Inequal. Appl.* **20**, 4 (2017), 1003–1016.
- [11] C.-P. CHEN, J.-W. ZHAO AND F. QI, *Three inequalities involving hyperbolically trigonometric functions*, *Octagon Math. Mag.* **12**, 2 (2004), 592–596.
- [12] L. DEBNATH, C. MORTICI AND L. ZHU, *Refinements of Jordan–Stečkin and Becker–Stark inequalities*, *Results Math.* **67** (2015), 207–215.
- [13] L. DEBNATH AND C. J. ZHAO, *New strengthened Jordan's inequality and its applications*, *Appl. Math. Lett.* **16** (2003), 557–560.
- [14] H.-F. GE, *New Sharp Bounds for the Bernoulli Numbers and Refinement of Becker-Stark Inequalities*, *J. Inequal. Appl.* **2012**, Article ID 137507, 7 pages.
- [15] W. D. JIANG AND H. YUN, *Sharpening of Jordan's inequality and its applications*, *J. Inequal. Pure Appl. Math.* **7**, 3 (2006), Article 102.

- [16] D. HE AND Y.-Z. HUANG, *The extension of Redheffer inequalities in trigonometric functions*, J. Foshan Univ. **30**, 6 (2012), 37–44 (in Chinese).
- [17] J.-L. LI AND Y.-L. LI, *On the Strengthened Jordan's Inequality*, J. Inequal. Appl. **2007**, Article ID 74328, 8 pages.
- [18] B. J. MALEŠEVIĆ, *One method for proving inequalities by computer*, J. Inequal. Appl. **2007**, Article ID 78691.
- [19] Y. NISHIZAWA, *Sharp Becker–Stark's type inequalities with power exponential functions*, J. Inequal. Appl. **2015** (2015) 402, <http://rd.springer.com/article/10.1186/s13660-015-0932-9/fulltext.html>.
- [20] A. Y. ÖZBAN, *A new refined form of Jordan's inequality and its applications*, Appl. Math. Lett. **19** (2006), 155–160.
- [21] R. REDHEFFER, *Problem 5642*, Amer. Math. Monthly **76** (1969), 422.
- [22] J. SÁNDOR AND B. A. BHAYO, *On an inequality of Redheffer*, Miskolc Math. Notes, **16**, 1 (1969), 475–482.
- [23] S. B. STEČKIN, *Some remarks on trigonometric polynomials*, Uspekhi Matematicheskikh Nauk, vol. 10, no. 1 (63), (1955) 159–166 (in Russian).
- [24] Z.-J. SUN AND L. ZHU, *Simple proofs of the Cusa–Huygens–type and Becker–Stark–type inequalities*, J. Math. Inequal. **7** (2013), 563–567.
- [25] J. P. WILLIAMS, *Solution of problem 5642*, Amer. Math. Monthly **76** (1969) 1153–1154.
- [26] S. H. WU, *Sharpness and generalization of Jordan's inequality and its application*, Taiwanese J. Math. **12** (2008), 325–336.
- [27] S. H. WU AND L. DEBNATH, *A new generalized and sharp version of Jordan's inequality and its applications to the improvement of the Yang Le inequality*, Appl. Math. Lett. **19** (2006), 1378–1384.
- [28] S. H. WU AND L. DEBNATH, *A new generalized and sharp version of Jordan's inequality and its applications to the improvement of the Yang Le inequality, II*, Appl. Math. Lett. **20** (2007), 532–538.
- [29] S. H. WU AND L. DEBNATH, *Jordan-type inequalities for differentiable functions and their applications*, Appl. Math. Lett. **21** (2008), 803–809.
- [30] S. H. WU AND H. M. SRIVASTAVA, *A further refinement of a Jordan type inequality and its application*, Appl. Math. Comput. **197** (2008), 914–923.
- [31] L. YANG, *Distribution of Values and New Research*, Science Press, Beijing, 1982 (in Chinese).
- [32] X. H. ZHANG, G. D. WANG AND Y. M. CHU, *Extensions and sharpenings of Jordan's and Kober's inequalities*, J. Inequal. Pure Appl. Math. **7**, 2 (2006), Article 63.
- [33] C. J. ZHAO, *Generalization and strengthen of Yang Le inequality*, Mathematics Practice Theory **30** (2000), 493–497 (in Chinese).
- [34] L. ZHU, *Sharpening Jordan's inequality and Yang Le inequality*, Appl. Math. Lett. **19** (2006), 240–243.
- [35] L. ZHU, *Sharpening Jordan's inequality and Yang Le inequality, II*, Appl. Math. Lett. **19** (2006), 990–994.
- [36] L. ZHU, *General forms of Jordan and Yang Le inequalities*, Appl. Math. Lett. **22** (2009), 236–241.
- [37] L. ZHU, *Sharpening Redheffer-type inequalities for circular functions*, App. Math. Lett. **22**, 5 (2009), 743–748,
- [38] L. ZHU, *Sharp Becker-Stark-type inequalities for Bessel functions*, J. Inequal. Appl. **2010**, Article ID 838740, 4 pages.
- [39] L. ZHU, *A refinement of the Becker-Stark inequalities*, Math. Notes **93** (2013), 421–425.
- [40] L. ZHU AND J. K. HUA, *Sharpening the Becker-Stark inequalities*, J. Inequal. Appl. **2010**, Article ID 931275, 4 pages.
- [41] L. ZHU AND J. SUN, *Six new Redheffer-type inequalities for circular and hyperbolic functions*, Comput. Math. Appl. **56**, 2 (2008) 522–529.