

## A SHARPENING OF A PROBLEM ON BERNSTEIN POLYNOMIALS AND CONVEX FUNCTIONS AND RELATED RESULTS

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*Abstract.* We present a short proof of a conjecture proposed by I. Raşa (2017), which is an inequality involving basic Bernstein polynomials and convex functions. This proof was given in the letter to I. Raşa (2017). The methods of our proof allow us to obtain some extended versions of this inequality as well as other inequalities given by I. Raşa. As a tool we use stochastic convex ordering relations. We propose also some generalizations of the binomial convex concentration inequality. We use it to insert some additional expressions between left and right sides of the Raşa inequalities.

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