

ON SOME CLASSICAL TRACE INEQUALITIES AND A NEW HILBERT–SCHMIDT NORM INEQUALITY

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Abstract. Let A be a positive semidefinite matrix and B be a Hermitian matrix. Using some classical trace inequalities, we prove, among other inequalities, that

$$\|A^s B + BA^{1-s}\|_2 \leq \|A^t B + BA^{1-t}\|_2$$

for $\frac{1}{2} \leq s \leq t \leq 1$. We conjecture that this inequality is also true for all unitarily invariant norms, and we affirmatively settle this conjecture for the case $s = \frac{1}{2}$ and $t = 1$.

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