

INTEGRAL INEQUALITIES OF LEVINSON'S TYPE IN TIME SCALE SETTINGS

JOSIPA BARIĆ, JOSIP PEČARIĆ AND DAJANA RADIŠIĆ

Abstract. A new class of functions, $\mathcal{K}_1^c(I)$, has been recently introduced by Baloch, Pečarić and Praljak. The authors proved that $\mathcal{K}_1^c(I)$ is the largest class of functions for which Levinson's inequality holds under Mercer's assumptions. We obtain Levinson's type inequalities in time scale settings by using the class $\mathcal{K}_1^c(I)$ and some known results regarding integral inequalities for convex (concave) functions on time scale sets.

Mathematics subject classification (2010): 26D15, 26A51.

Keywords and phrases: Levinson's inequality, Jensen's inequality, time scale calculus.

REFERENCES

- [1] S. ABRAMOVICH, J. BARIĆ, J. PEČARIĆ, *A variant of Jensen's inequality of Mercer's type for superquadratic functions*, J. Inequal. Pure Appl. Math., **9**, 3 (2008).
- [2] R. P. AGARWAL, M. BOHNER, A. PETERSON, *Inequalities on time scale: a survey*, Mathematical Inequalities and Applications, **4**, 4, (2001), 535–557.
- [3] M. ANWAR, R. BIBI, M. BOHNER, J. PEČARIĆ, *Integral inequalities on time scale via the theory of isotonic linear functionals*, Abstract and Applied Analysis, **2011**, article id 483595, 16pp.
- [4] I. A. BALOCH, J. PEČARIĆ, M. PRALJAK, *Generalization of Levinson's inequality*, J. Math. Inequal., **9**, 2 (2015), 571–586.
- [5] J. BARIĆ, M. BOHNER, R. JAKŠIĆ, J. PEČARIĆ, *Converses of Jensen's inequality on time scales*, Mathematical Notes, **98**, 1 (2015), 11–24.
- [6] J. BARIĆ, R. BIBI, M. BOHNER, A. NOSHEEN, J. PEČARIĆ, *Jensen Inequalities on Time Scales, Theory and Applications*, Zagreb, Element, 2015., monograph in inequalities.
- [7] M. BOHNER AND G. SH. GUSEINOV, *Multiple integration on time scales*, Dynam. Systems Appl. **14** 3-4, (2005) 579–606.
- [8] M. BOHNER AND G. SH. GUSEINOV, *Multiple Lebesgue integration on time scales*, Adv. Difference Equ., 2006, Art. ID 26391, 12 pp.
- [9] M. BOHNER, A. PETERSON, *Dynamic Equations on Time Scales*, Birkhäuser, Boston, Basel, Berlin, 2001.
- [10] P. S. BULLEN, *An inequality of N. Levinson*, Univ. Beograd Publ. Elektrotehn. Fak. Ser. Mat. Fiz. **421-460** (1973), 109–112.
- [11] W. S. CHEUNG, A. MATKOVIĆ, AND J. PEČARIĆ, *A variant of Jensen's inequality and generalized means*, J. Inequal. Pure Appl. Math., **7** 1, (2006).
- [12] S. HILGER, *Ein Maßkettenkalkül mit Anwendung auf Zentrumsmannigfaltigkeiten*, Phd. D. thesis, Universität Würzburg, 1988.
- [13] S. HILGER, *Analysis on measure chains -a unified approach to continuous and discrete calculus*, Results Math. **18**, 1–2 (1990), 18–56.
- [14] S. HILGER, *Differential and difference calculus unified*, Nonlinear Anal. **30**, 1 (1997), 143–166.
- [15] B. KAYMAKÇALAN, V. LAKSHMIKANTHAM, S. SIVASUNDARAM, *Dynamic Systems on Measure Chains*, Kluwer Academic Publishers, Dordrecht, 1996.
- [16] N. LEVINSON, *Generalization of an inequality of Ky Fan*, J. Math. Anal. Appl. **8** (1964), 133–134.
- [17] A. MCD. MERCER, *A variant of Jensen's inequality*, J. Inequal. Pure Appl. Math., **4** 4, article 73, 2 pp. (electronic), 2003.

- [18] A. MCD. MERCER, *Short proof of Jensen's and Levinson's inequalities*, Math. Gazette **94** (2010), 492–495.
- [19] J. PEČARIĆ, M. PRALJAK, A. WITKOWSKI, *Generalized Levinson's inequality and exponential convexity*, Opuscula Math. **35**, 3 (2015), 397–410.
- [20] T. POPOVICIU, *Sur une inegalite de N. Levinson*, Mathematica (Cluj) **6** (1964), 301–306.
- [21] A. WITKOWSKI, *On Levinson's inequality*, RGMIA Research Report Collection **15** (2012), Article 68.
- [22] F.-H. WONG, C.-C. YEH, W.-C. LIAN, *An extension of Jensen's inequality on time scales*, Advances in Dynamical Systems and Applications, **1**, 1, (2006), 113–120.