

## EQUALITY CASES OF INEQUALITIES INVOLVING GENERALIZED CSISZÁR AND TSALLIS TYPE $f$ -DIVERGENCES

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*Abstract.* In this note, we study the problem of equality case of two inequalities involving generalized Csizsár  $f$ -divergences and generalized Tsallis  $f$ -divergences, respectively, with a convex function  $f$ . To this end we use generalized inverses of matrices and inverse-positive matrices.

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