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THE FEFFERMAN–STEIN TYPE INEQUALITIES FOR THE MULTILINEAR STRONG MAXIMAL FUNCTIONS

JUAN ZHANG, HIROKI SAITO AND QINGYING XUE

Abstract. Let $\vec{\omega} = (\omega_1, \dots, \omega_m)$ be a multiple weight and $\{\Psi_j\}_{j=1}^m$ be a sequence of Young functions. Let $\mathscr{M}_{\mathscr{R}}^{\vec{\Psi}}$ be the multilinear strong maximal function with Orlicz norms which is defined by

$$\mathscr{M}_{\mathscr{R}}^{\vec{\Psi}}(\vec{f})(x) = \sup_{R \in \mathscr{R}, R \ni x} \prod_{j=1}^{m} \|f_j\|_{\Psi_j, R},$$

where the supremum is taken over all rectangles with sides parallel to the coordinate axes. If $\Psi_j(t) = t$, then $\mathscr{M}_{\mathscr{R}}^{\vec{t}}$ coincides with the multilinear strong maximal function $\mathscr{M}_{\mathscr{R}}$ defined and studied by Grafakos et al. In this paper, we first investigated the Fefferman-Stein type inequality for $\mathscr{M}_{\mathscr{R}}^{\vec{\Psi}}$ when $\vec{\omega}$ satisfies the $A_{\infty,\mathscr{R}}$ condition. Then, for arbitrary $\vec{\omega} \geqslant 0$ (each $\omega_j \geqslant 0$), the Fefferman-Stein type inequality for the multilinear strong maximal function $\mathscr{M}_{\mathscr{R}}$ associated with rectangles will be given.

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