

HAUSDORFF OPERATOR IN LEBESGUE SPACES

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Abstract. We study the boundedness of the Hausdorff operator in various types of Lebesgue spaces, e.g. weighted spaces, variable exponent and grand Lebesgue spaces. The results are illustrated by a number of examples.

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