

## BILINEAR FOURIER MULTIPLIER OPERATORS ON VARIABLE TRIEBEL SPACES

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*Abstract.* In this paper, we prove the boundedness of bilinear Fourier multiplier operators on variable exponent Triebel-Lizorkin spaces.

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