

ROUGH FRACTIONAL INTEGRAL OPERATORS AND BEYOND ADAMS INEQUALITIES

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Abstract. We consider the boundedness of fractional integral operators with rough kernel from Morrey spaces $L^{p,\lambda}$ to $L^{q,\mu}$. Our main concern is proving the boundedness property for $\mu < \lambda$ as an extension of Adams inequality on some special subsets of the operator's domain namely classes of A_p , simple function, and radial function respectively. For radial function, we prove the boundedness on local Morrey spaces. We also prove the boundedness property for $\mu \geq \lambda$ as well as the special case of $q \leq p$. It is interesting on its own term since the operator is not bounded from L^p to L^q if $q \leq p$. We also establish necessary conditions for boundedness. Our proposed condition for boundedness includes the sufficient conditions for both Adams inequality and Spanne inequality.

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