

COMMUTATORS OF POTENTIAL TYPE OPERATORS WITH LIPSCHITZ SYMBOLS ON VARIABLE LEBESGUE SPACES WITH DIFFERENT WEIGHTS

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Abstract. We prove that a generalized Fefferman-Phong type condition on a pair of weights u and v is sufficient for the boundedness of the commutators of potential type operators from $L_v^{p(\cdot)}$ into $L_u^{q(\cdot)}$. We also give an improvement of this result in the sense that we not only consider a variable version of power bump conditions, but also weaker norms related to Musielak-Orlicz functions.

We consider a wider class of symbols including Lipschitz symbols and some generalizations.

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