

## QUANTITATIVE WEIGHTED $L^p$ BOUNDS FOR THE MARCINKIEWICZ INTEGRAL

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*Abstract.* Let  $\Omega$  be homogeneous of degree zero, have mean value zero and integrable on the unit sphere, and  $\mu_\Omega$  be the higher-dimensional Marcinkiewicz integral associated with  $\Omega$ . In this paper, the authors proved that if  $\Omega \in L^q(S^{n-1})$  for some  $q \in (1, \infty]$ , then for  $p \in (q', \infty)$  and  $w \in A_p(\mathbb{R}^n)$ , the bound of  $\mu_\Omega$  on  $L^p(\mathbb{R}^n, w)$  is less than  $C[w]_{A_p/q'}^{\max\{\frac{1}{2}, \frac{1}{p-q'}\} + \max\{1, \frac{q'}{p-q}\}}$ .

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