

MONOTONICITY PATTERNS AND FUNCTIONAL INEQUALITIES FOR CLASSICAL AND GENERALIZED WRIGHT FUNCTIONS

KHALED MEHREZ

Abstract. Our aim in this paper is to present the completely monotonicity and convexity properties for the Wright function. As a consequences of these results, we present some functional inequalities. Moreover, we derive the monotonicity and log-convexity results for the generalized Wright functions. As applications, we present several new inequalities (like Turán type inequalities) and we prove some geometric properties for the four-parametric Mittag–Leffler functions.

Mathematics subject classification (2010): 33C20, 33E12, 26D07.

Keywords and phrases: Wright function, generalized Wright function, four-parametric Mittag–Leffler function, complete monotonicity, Turán type inequalities.

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