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ESTIMATIONS OF THE WEIGHTED POWER MEAN BY THE HERON MEAN AND RELATED INEQUALITIES FOR DETERMINANTS AND TRACES

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Abstract. For positive real numbers a and b, the weighted power mean $P_{t,q}(a,b)$ and the weighted Heron mean $K_{t,q}(a,b)$ are defined as follows: For $t \in [0,1]$ and $q \in \mathbb{R}$, $P_{t,q}(a,b) = \{(1-t)a^q + tb^q\}^{\frac{1}{q}}$ and $K_{t,q}(a,b) = (1-q)a^{1-t}b^t + q\{(1-t)a + tb\}$. These means generalize the arithmetic and geometric ones.

In this paper, as a generalization of Wu and Debnath's result on non-weighted means (the case $t = \frac{1}{2}$), we get estimations of the weighted power mean by the weighted Heron mean. In other words, we obtain the greatest value $\alpha_1 = \alpha_1(t,r)$ and the least value $\alpha_2 = \alpha_2(t,r)$ such that the double inequality $K_{t,\alpha_1}(a,b) < P_{t,r}(a,b) < K_{t,\alpha_2}(a,b)$ holds for $t \in (0,1)$ and $r \in \mathbb{R}$. We can also obtain the results for bounded linear operators on a Hilbert space. Moreover, our main results lead some determinant and trace inequalities of matrices.

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