

## GENERALIZED STEFFENSEN'S INEQUALITY BY MONTGOMERY IDENTITIES AND GREEN FUNCTIONS

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*Abstract.* A new generalization of Steffensen's inequality and other inequalities related to Steffensen's inequality have been proved. The contribution of these new generalizations has been presented to theory of  $(n + 1)$ -convex functions and exponentially convex functions.

*Mathematics subject classification (2010):* Primary 26D10, 26D15, Secondary 26D20.

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