

PÓLYA-SZEGŐ AND CHEBYSHEV TYPES INEQUALITIES VIA AN EXTENDED GENERALIZED MITTAG-LEFFLER FUNCTION

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Abstract. In this paper certain Pólya-Szegő type integral inequalities due to Karamata's estimations of the Chebyshev quotient are presented. Those inequalities include an extended generalized Mittag-Leffler function with the corresponding fractional integral operator, and from them, some fractional integral inequalities of Chebyshev type are obtained. Also, several known results are improved.

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