

LOCAL SHARP MAXIMAL FUNCTIONS, GEOMETRICAL MAXIMAL FUNCTIONS AND ROUGH MAXIMAL FUNCTIONS ON LOCAL MORREY SPACES WITH VARIABLE EXPONENTS

TAT-LEUNG YEE, KA LUEN CHEUNG, KWOK-PUN HO AND CHUN KIT SUEN

Abstract. We study the local Morrey spaces with variable exponents. We show that the local block space with variable exponents are pre-duals of the local Morrey spaces with variable exponents. Using this duality, we establish the extrapolation theory for the local Morrey spaces with variable exponents. The extrapolation theory gives the mapping properties for the local sharp maximal functions, the geometric maximal functions and the rough maximal function on the local Morrey spaces with variable exponents.

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