

THE OPTIMAL CONSTANT IN GENERALIZED HARDY'S INEQUALITY

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Abstract. We obtain the sharp factor of the two-sides estimates of the optimal constant in generalized Hardy's inequality with two general Borel measures on \mathbb{R} , which generalizes and unifies the known continuous and discrete cases.

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