

INCLUSION BETWEEN GENERALIZED STUMMEL CLASSES AND OTHER FUNCTION SPACES

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Abstract. We refine the definition of generalized Stummel classes and study inclusion properties of these classes. We also study the inclusion relation between Stummel classes and other function spaces such as generalized Morrey spaces, weak Morrey spaces, and Lorentz spaces. In addition, we show that these inclusions are proper. Our results extend some previous results in [2, 13].

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