Mathematical
Inequalities
& Applications
Volume 23, Number 2 (2020), 611–624 doi:10.7153/mia-2020-23-50

A NEW FRACTIONAL ORDER POINCARE'S INEQUALITY WITH WEIGHTS

Farman Mamedov, Nazira Mammadzade and Lars-Erik Persson

Abstract. We derive a new Sawyer's type sufficient condition for the fractional order Poincare inequality with weights

$$\Big(\int_{\Omega} |f(x)-\overline{f}_{v,\Omega}|^q\, v(x)\,dx\Big)^{\frac{1}{q}} \leqslant C\,\Big(\iint_{\Omega\times\Omega} |f(x)-f(y)|^p\, \omega(x,y)\,dxdy\Big)^{\frac{1}{p}}$$

to hold in a non-regular domain $\Omega\subset R^n$ of finite volume, where $\omega(x,y)=|x-y|^{-n-\alpha p}\omega_0(x,y)$, $0<\alpha<1, q\geqslant p>1$, $f\in C(\Omega)$, and $v(\cdot)$, $\omega(\cdot,\cdot)$ are positive measurable functions such that $\omega^{1-p'}(x,\cdot)\,v^{p'}(\cdot)\in L(\Omega)$ a.e. $x\in\Omega$ and $\overline{f}_{v,\Omega}=\frac{1}{v(\Omega)}\int_{\Omega} fv\,dx$.

Mathematics subject classification (2010): 26D10, 35A23, 35J15, 35J70, 46E35.
Keywords and phrases: Inequalities, fractional Hardy-Sobolev's inequality, weights, Poincare's inequality, fractional order Poincare inequality.

REFERENCES

[1] R. Amanov and F. Mamedov, *On some properties of solutions of quasilinear degenerate equations*, Ukrain. Math. J., **60**, 7 (2008), 918–936.
[2] R. Amanov and F. Mamedov, *Regularity of the solutions of degenerate elliptic equations in divergent form*, Math. Notes, **83**, 1&2 (2008), 3–13.
[3] K. Bogdan and B. Dyda, *The best constant in a fractional Hardy inequality*, Math. Nachr., **284**, 5–6 (2011), 629–638.
[4] L. Brasco and E. Cinti, *On fractional Hardy inequalities in convex sets*, arXiv: 1802.02354, 2018.
[5] J. Bourgain, H. Brezis, and P. Mironescu, *Limiting embedding theorems for $W^{s,p}$ when $s\uparrow 1$ and applications*, J. Anal. Math., **87** (2002), 77–101.
[6] V. I. Burenkov and W. D. Evans, *Weighted Hardy-type inequalities for differences and the extension problem for spaces with generalized smoothness*, J. London Math. Soc., **57**, 2 (1998), 209–230.
[7] V. I. Burenkov, W. D. Evans, and M. L. Goldman, *On weighted Hardy and Poincare-type inequalities for differences*, J. Inequal. Appl., **1**, 1 (1997), 1–10.
[8] D. E. Edmunds, R. H. Syrjanen, and A. V. Vahakangas, *Fractional Hardy-type inequalities in domains with uniformly fat complement*, Proc. Amer.Math.Soc., **142**, 3 (2014), 897–907.
[9] Z. Q. Chen and R. Song, *Hardy inequality for censored stable processes*, Tohoku Math. J., **55**, 2 (2003), 439–450.
[10] B. L. Dyda, *A fractional order Hardy inequality*, Illinois J. Math., **48**, 2(2004), 575–588.
[11] B. Dyda, L. Ihnatsyeva and A. Vahakangas, *On improved fractional Sobolev-Poincare inequalities*, Ark. Mat., **54**, 2(2016), 437–457.
[12] B. Dyda and A. V. Vahakangas, *Characterizations for fractional Hardy inequality*, Adv. Calc. Var., **8** (2015), 173–182, 2015.
[13] B. Dyda and A. V. Vahakangas, *A framework for fractional Hardy inequalities*, Ann. Acad. Sci. Fenn. Math., **39**, 2(2014), 675–689.
[14] R. L. Frank and R. Seiringer, *Non-linear ground state representations and sharp Hardy inequalities*, J. Funct. Anal., **255**, 12(2008), 3407–3430.

Paper MIA-23-50

[15] D. GILBARG AND N. S. TRUDINGER, *Elliptic partial differential equations of second order*, Springer, 2001, 517pp.
[16] P. GURKA AND B. OPIC, *Sharp Hardy inequalities of fractional order involving slowly varying functions*, J. Math. Anal. Appl., **386**, 2(2012) 728–737.
[17] P. GRISVARD, *Espaces intermediaires entre espaces de Sobolev avec poids*, Ann. Scuola Norm. Sup. Pisa, **17**, 3(1963), 255–296.
[18] H. P. HEINIG, A. KUFNER AND L.-E. PERSSON, *On some fractional order Hardy inequalities weighted nonlinear potential theory*, J.Inequal. Appl. **1**(1997), 25–46.
[19] L. IHNATSYEVA, J. LEHRBACK, H. TUOMINEN AND A. VAHAKANGAS, *Fractional Hardy inequalities and visibility of the boundary*, Studia Math., **224**, 1(2014), 47–80.
[20] G. N. JAKOVLEV, *Boundary properties of functions of the class $W_p^{(l)}$ in regions with corners*, Dokl. Akad. Nauk USSR, **140**(1961), 73–76.
[21] P. KIM AND A. MIMICA, *Harnack inequalities for subordinate Brownian motions*, Electron. J. Prob., **17**, 37(2012), 23.
[22] N. KRUGLYAK, L. MALIGRANDA AND L.-E. PERSSON, *On an elementary approach to the fractional Hardy inequality*, Proc. Amer. Math. Soc., **128**(1999), 727–734.
[23] A. KUFNER AND L.-E. PERSSON, *Integral inequalities with weights*, Academia, Prague, 2000.
[24] A. KUFNER AND L.-E. PERSSON, *Some difference inequalities with weights and interpolation*, Math. Inequal. Appl, **1**, 3(1998), 437–444.
[25] A. KUFNER, L.-E. PERSSON AND N. SAMKO, *Weighted Inequalities of Hardy Type*, Second Edition, World Scientific, New Jersey, 2017, 479 pp.
[26] A. KUFNER AND H. TRIEBEL, *Generalizations of Hardy's inequality*, Conf. Sem. Mat. Univ. Bari, **156**(1978), 1–21.
[27] F. I. MAMEDOV, *On the multidimensional weighted Hardy inequalities of fractional order*, Proc. Inst. of Math. Mech. Acad. Sci. Azerb., **10**(1999), 102–114.
[28] F. MAMEDOV, *On some weighed inequalities of the qualitative theory of elliptic equations*, Bulletin of TICMI, **4**(2000), 31–35 (Advanced Course on Function Spaces and Applications. Papers of Mini symposium).
[29] F. I. MAMEDOV, *On the Harnack inequality for an equation that is formally conjugate to a linear elliptic differential equation*, Siberian Math. J., **33**, 5(1992), 835–841.
[30] F. MAMEDOV AND R. AMANOV, *On some nonuniform cases of the weighted Sobolev and Poincare inequalities*, St. Petersburg Math. J.(Algebra i Analize), **20**, 3(2009), 447–463.
[31] F. MAMEDOV AND Y. SHUKUROV, *A Sawyer-type sufficient condition for the weighted Poincare inequality*, Positivity, **22**, 3(2018), 687–699.
[32] V. G. MAZYA AND T. O. SHAPOSHNIKOVA, *Erratum to "On the Bourgain, Brezis, and Mironescu theorem concerning limiting embeddings of fractional Sobolev spaces"*, J. Funct. Anal., **201**(2003), 298–300.
[33] V. G. MAZYA AND T. O. SHAPOSHNIKOVA, *On the Bourgain, Brezis, and Mironescu theorem concerning limiting embeddings of fractional Sobolev spaces*, J. Funct. Anal., **195**, 4(2002), 230–238.
[34] J. NECAS, *Sur une methode pour resoudre les equations aux derivees partielles du type elliptique, voisine de la variationnelle*, Ann. Scuola Norm. Sup. Pisa, **16**, 3(1962), 305–326.
[35] M. DE QUZMAN, *Differentation of integrals in R^n*, Springer, **481**, Lecture Notes in Math., Springer Verlag, Berlin, New York, 1975, 228 pp.
[36] H. ŠIKIĆ, R. SONG, AND Z. VONDRAČEK, *Potential theory of geometric stable processes*, Probab. Theory Related Fields, **135**, 4(2006), 547–575.
[37] R. HURRI-SYRJANEN AND A. V. VAHAKANGAS, *Fractional Sobolev-Poincare and fractional Hardy inequalities in unbounded John domains*, Mathematika, **61**, 2(2015), 385–401.
[38] R. HURRI-SYRJANEN AND A. V. VAHAKANGAS, *On fractional Poincare inequalities*, J. Anal. Math., **120**(2013), 85–104.
[39] L. SIROTA AND E. OSTROVSKY, *Necessary conditions for fractional Hardy-Sobolev's Inequalities*, ArXive: 1108.1387v1 [math. FA], 5 Aug 2011.
[40] H. TRIEBEL, *Interpolation theory, function spaces, differential operators*, Johann Ambrosius Barth. Verlag, Heidelberg, Leipzig, 1998, 532 pp.
[41] Y. ZHOU, *Fractional Sobolev extension and imbedding*, Trans.Amer. Math.Soc., **367**(2015), 959–979.