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ON A GENERALIZED EGNELL INEQUALITY

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Abstract. In this paper we prove an inequality which connects the L^p norm of the gradient of a function u with its $|x|^\nu$-weighted $L^{\frac{p(N+\nu)}{N-p}}$ norm and its L^{p^*}-weak norm. Here $1 < p < N$, $-p < \nu \leqslant 0$ and $p^* = \frac{Np}{N-p}$. As a consequence we can provide an alternative proof of the Egnell inequality in $\mathbb{R}^N$.

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