

## BOUNDEDNESS AND COMPACTNESS OF THE HARDY TYPE OPERATOR WITH VARIABLE UPPER LIMIT IN WEIGHTED LEBESGUE SPACES

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*Abstract.* Let  $0 < \alpha < 1$ . The operator of the form

$$K_{\alpha, \varphi} f(x) = \int_a^{\varphi(x)} \frac{f(t)w(t)dt}{(W(x) - W(t))^{(1-\alpha)}}, \quad x > 0,$$

is considered, where the real weight functions  $v(x)$  and  $w(x)$  are locally integrable on  $I := (a, b)$ ,  $0 \leq a < b \leq \infty$  and  $\frac{dW(x)}{dx} \equiv w(x)$ . In this paper we derive criteria for the operator  $K_{\alpha, \varphi}$ ,  $0 < \alpha < 1$ ,  $0 < p, q < \infty$ ,  $p > \frac{1}{\alpha}$  to be bounded and compact from the spaces  $L_{p, w}$  to the spaces  $L_{q, v}$ .

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### REFERENCES

- [1] A.M. ABYLAYEVA, *Criterion of the boundedness of  $0 < \alpha < 1$  - order fractional integration type operator*, Bulletin of L.N. Gumilyov ENU, **46**, 6 2005, 130–136 (in Russian).
- [2] A.M. ABYLAYEVA, *Inequalities for Some Classes of Hardy Type Operators and Compactness in Weighted Lebesgue Spaces*, Doctoral Thesis, Luleå University of Technology, Sweden, ISSN 1402-1544, December 2016.
- [3] A.M. ABYLAYEVA AND R. OINAROV, *Criterion of the boundedness of a class of fractional integration operators*, Math. J. Almaty, **4**, 12 (2004), No.2, 5–14 (in Russian).
- [4] D.E. EDMUNDS, V. KOKILASHVILI AND A. MESKHI, *Bounded and compact integral operators*, Kluwer Academic Published, Boston / Dordrecht / London, 2002.
- [5] L.V. KANTAROVICH AND G.R. AKILOV, *Functional analysis*, Publishing House "Nauka", Moscow, 1977, 741 (in Russian).
- [6] M.A. KRASNOSELSKII, P.P. ZABREIKO, E.I. PUSTYLNİK AND P.E. SOBOLEVSKII, *Integral operators in space of summable functions*, Publishing House "Nauka", Moscow, 1966, 500 (in Russian).
- [7] A. KUFNER, L. MALIGRANDA AND L.-E. PERSSON, *The Hardy Inequality. About its History and some Related Results*, Vydavatelský Servis Publishing House, Pilsen, 2007.
- [8] A. KUFNER, L.-E. PERSSON AND N.SAMKO, *Weighted Inequalities of Hardy Type*, Second Edition, World Scientific, New Jersey-London-Singapore-Beijing-ShanghaiHong Kong-Taipei-Chennai-Tokyo, 2017.
- [9] M.A. LORENTE, *A characterization of two weight norm inequalities for one-side operators of fractional type*, Can.Math.J. **49**, 5 (1997), 1010–1033.
- [10] A. MESKHI, *Solution of some weight problems for the Reimann -Liouville and Weyl operators*, Georgian Math.J. **5**, 6 (1998), 565–574.
- [11] D.V. PROKHOROV, *On the boundedness and compactness of a class of integral operators*, J.London Math. Soc. **61**, 2 (2000), 617–628.
- [12] D.V. PROKHOROV AND V.D. STEPANOV, *Riemann-Liouville operators*, Reports of the Russian Academy of Sciencesh. **382**, 4 (2002), 452–455 (in Russian).