

SHARP LOWER AND UPPER BOUNDS FOR THE q -GAMMA FUNCTION

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Abstract. This paper is devoted to provide sharp bounds for the q -gamma function from below and above for all $q > 0$ by means of investigating the monotonicity property to analytical functions involving logarithm q -gamma function. It turns out that these results refine and improve lower and upper bounds for the q -gamma function which have been given by Salem [13].

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