

SOME ESTIMATES FOR THE BILINEAR FRACTIONAL INTEGRALS ON THE MORREY SPACE

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Abstract. In this paper, we are interested in the following bilinear fractional integral operator $B\mathcal{I}_\alpha$ defined by

$$B\mathcal{I}_\alpha(f, g)(x) = \int_{\mathbb{R}^n} \frac{f(x-y)g(x+y)}{|y|^{n-\alpha}} dy,$$

with $0 < \alpha < n$. We prove the weighted boundedness of $B\mathcal{I}_\alpha$ on the Morrey type spaces. Moreover, an Olsen type inequality for $B\mathcal{I}_\alpha$ is also given.

Mathematics subject classification (2010): 42B20, 42B25.

Keywords and phrases: Weighted norm inequalities, bilinear fractional integral, dyadic grids, Morrey spaces, Olsen type inequalities.

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