

FUNDAMENTAL HLAWKA-LIKE INEQUALITIES FOR THREE AND FOUR VECTORS

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Abstract. We investigate Hlawka-like inequalities for three vectors and determine necessary and sufficient conditions such that

$$a_1 \sum_{i=1}^3 \|x_i\| + a_2 \sum_{1 \leq i < j \leq 3} \|x_i + x_j\| + a_3 \|x_1 + x_2 + x_3\| \geq 0$$

is satisfied for all x_1, x_2, x_3 in a Hlawka space. In addition, we show that any such inequality can be obtained as a linear combination with nonnegative coefficients of three fundamental inequalities, one of which is Hlawka's inequality.

In the case of four vectors in an inner product space, we prove that any (valid) inequality of the form

$$a_1 \sum_{i=1}^4 \|x_i\| + a_2 \sum_{1 \leq i < j \leq 4} \|x_i + x_j\| + a_3 \sum_{1 \leq i < j < k \leq 4} \|x_i + x_j + x_k\| + a_4 \left\| \sum_{i=1}^4 x_i \right\| \geq 0$$

can be written as a linear combination with nonnegative coefficients of six fundamental inequalities.

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REFERENCES

- [1] D. D. ADAMOVIĆ, *Généralisation d'une identité de Hlawka et de l'inegalite correspondante*, Mat. Vesnik 1 **16** (1964), 39–43.
- [2] W. BERNDT, S. SRA, *Hlawka-Popoviciu inequalities on positive definite tensors*, Linear Algebra Appl. **486** (2015), 317–327.
- [3] D. Z. DJOKOVIĆ, *Generalizations of Hlawka's Inequality*, Glas. Mat.-Fiz. Astronom., Ser II, Društvo Mat. Fiz. Hratske, **18** (1963), 169–175.
- [4] W. FECHNER, *Hlawka's functional inequality*, Aequationes Math. **87** (2014), no. 1-2, 71–87.
- [5] L. M. KELLY, D. M. SMILEY, M. F. SMILEY, *Two dimensional spaces are quadrilateral spaces*, Am. Math. Month. **72** (1965), 753–754.
- [6] M. MUNTEANU, *Extensions of Hlawka's inequality for four vectors*, J. Math. Inequal. **13** (2019), no. 3, 891–901.
- [7] D. S. MITRINOVIĆ, *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.
- [8] D. M. SMILEY, M. F. SMILEY, *The polygonal inequalities*, Amer. Math. Monthly **71** (1964), 755–760.
- [9] A. SIMON, P. VOLKMANN, *On Two Geometric Inequalities*, Ann. Math. Sil. **9** (1995), 137–140.
- [10] TAKAHASHI, Y., WADA, S., TAKAHASHI, S.-E., *An extension of Hlawka's inequality*, Math. Inequal. Appl. **3**(1) (2000), 63–67.
- [11] WITSENHAUSEN, H. S., *Metric inequalities and the zonoid problem*, Proc. Am. Math. Soc. **40** (1973), 517–520.