

VOLTERRA INTEGRAL OPERATORS FROM \mathcal{D}_{p-2+s}^p INTO $F(p\lambda, p\lambda + s\lambda - 2, q)$

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Abstract. Let $1 < p < \infty$, $0 < q < \infty$, $0 < s$, $\lambda \leq 1$ such that $q + s\lambda > 1$. We characterize the boundedness and compactness of inclusion mapping from Dirichlet type spaces \mathcal{D}_{p-2+s}^p into tent spaces $T_{p\lambda, q}(\mu)$. As an application, the boundedness of the Volterra operator T_g , its companion operator I_g and the multiplication operator M_g from \mathcal{D}_{p-2+s}^p to $F(p\lambda, p\lambda + s\lambda - 2, q)$ are given. Furthermore, we study the essential norm and compactness of T_g and I_g .

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