

GENERALIZED TRIANGLE INEQUALITY OF THE SECOND TYPE IN QUASI NORMED SPACES

ASIYEH REZAEI AND FARZAD DADIPOUR

Abstract. We investigate a generalized triangle inequality of the second type in the framework of quasi normed spaces. More precisely, by using the well-known Aoki-Rolewicz theorem and some quasi normed inequalities, we obtain some regions of \mathbb{R}^n which contain the set of all n -tuples satisfying the mentioned inequality. Moreover, some reverse inclusions are also discussed. As applications, we deduce some new results associated with generalizations of the triangle inequality in p -normed spaces and we get some already known results in a new approach.

Mathematics subject classification (2010): 46A16, 47A30, 46B20.

Keywords and phrases: Triangle inequality of the second type, generalized triangle inequality, Aoki-Rolewicz theorem, quasi normed space.

REFERENCES

- [1] T. AOKI, *Locally bounded topological spaces*, Proc. Imp. Acad. Tokyo, **18**, (1942), 588–594.
- [2] H. BELBACHIR, M. MIRZAVAZIRI AND M. S. MOSLEHIAN, *q -norms are really norms*, Aust. J. Math. Anal. Appl., **3**, (2006), 1–3.
- [3] F. DADIPOUR, M. S. MOSLEHIAN, J. M. RASSIAS AND S. E. TAKAHASI, *Characterization of a generalized triangle inequality in normed spaces*, Nonlinear Anal., **75**, (2012), 735–741.
- [4] S. S. DRAGOMIR, Y. J. CHO AND S. S. KIM, *Some inequalities in inner product spaces related to the generalized triangle inequality*, Appl. Math. Comput., **217**, (2011), 5.
- [5] H. HUDZIK AND T. R. LANDES, *Characteristic of convexity of Köthe function spaces*, Math. Ann., **294**, (1992), 117–124.
- [6] R. MALČESKI, *Sharp triangle inequalities in quasi-normed spaces*, British J. Math. Comput. Sci., **5**, (2015), 258–265.
- [7] L. ARAMBAŠIĆ AND R. RAJIĆ, *On the C^* -valued triangle equality and inequality in Hilbert C^* -modules*, Acta Math. Hungar., **119**, (2008), 373–380.
- [8] N. MINCULETE AND R. PĂLTĂNEA, *Improved estimates for the triangle inequality*, J. Inequal. Appl., **17**, (2017), 1–12.
- [9] T. IZUMIDA, K. I. MITANI AND K. S. SAITO, *Another approach to characterizations of generalized triangle inequalities in normed spaces*, Cent. Eur. J. Math., **12**, (2014), 1615–1623.
- [10] N. J. KALTON, N. T. PECK AND J. W. ROBERTS, *An F -space sampler*, London Math. Soc. **89**, Cambridge University Press, 1984.
- [11] A. E. LITVAK, *The extension of the finite-dimensional version of Krivine's theorem to quasi-normed spaces*, Convex Geometric Analysis, Math. Sci. Res. Inst. Publ., **34**, (1998), 139–148.
- [12] A. PIETSCH, *History of Banach spaces and linear operators*, Springer, Birkhäuser Publisher, 2007.
- [13] J. M. RASSIAS, *Solutions of the Ulam stability problem for Euler-Lagrange quadratic mappings*, J. Math. Anal. Appl., **220**, (1998), 613–639.
- [14] S. SAITOH, *Generalizations of the triangle inequality*, J. Inequal. Pure Appl. Math., **4**, (2003), 5.
- [15] S. E. TAKAHASI, J. M. RASSIAS, S. SAITOH AND Y. TAKAHASHI, *Refined generalizations of the triangle inequality on Banach space*, Math. Inequal. Appl., **13**, (2010), 733–741.
- [16] C. WU AND Y. LI, *On the triangle inequality in quasi-Banach spaces*, J. Inequal. Pure Appl. Math., **9**, (2008), 9.