

EULER-LAGRANGE EQUATIONS ASSOCIATED WITH EXTREMAL FUNCTIONS OF SEVERAL NONLOCAL INEQUALITIES

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Abstract. This paper is concerned with the extremal functions of several kinds of non-local inequalities, including the Hardy-Littlewood-Sobolev inequality, fractional Gagliardo-Nirenberg inequality, nonlocal Gagliardo-Nirenberg inequality and Coulomb-Sobolev inequality. First, we derive the Euler-Lagrange equations which they satisfy. Second, we investigate the existence of some integrable classical solutions for these equations, where the Pohozaev identity plays a key role.

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