

## TRANSFERENCE METHOD FOR CONE-LIKE RESTRICTED SUMMABILITY OF THE TWO-DIMENSIONAL WALSH-LIKE SYSTEMS

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*Abstract.* In the present paper we investigate the boundedness of the maximal operator of some  $d$ -dimensional means, provided that the set of the indeces is inside a cone-like set  $L$ . Applying some assumptions on the summation kernels  $P_{n_1, \dots, n_d}$  we state that the cone-like restricted maximal operator  $T_{CLR}^\gamma$  is bounded from the Hardy space  $H_p^\gamma$  to the Lebesgue space  $L^p$  for  $p > p_0$ . In the end point  $p_0$  assuming some natural conditions on one-dimensional kernels we show that the maximal operator  $T_{CLR}^\gamma$  is not bounded from the Hardy space  $H_{p_0}^\gamma$  to the Lebesgue space  $L^{p_0}$ .

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