

## A NOTE ON INTEGRAL REPRESENTATION OF SOME GENERALIZED ZETA FUNCTIONS AND ITS CONSEQUENCES

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*Abstract.* The main focus of the present note is to establish new integral representation for the Hurwitz-Lerch zeta and the multi-parameter Hurwitz-Lerch zeta functions. In particular, new integral expression of the polylogarithm function and the Fox-Wright function are derived. In addition, closed integral form expression of the moment generating function of a zeta distribution is established. As application, we derive the complete monotonicity properties of two classes of function related to the Hurwitz-Lerch zeta and the polylogarithm function. Moreover, some inequalities involving these two functions are proved.

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