

HOW THE TYPE OF CONVEXITY OF THE CORE FUNCTION AFFECTS THE CSISZÁR f -DIVERGENCE FUNCTIONAL

MOHSEN KIAN

Abstract. We investigate how the type of convexity of the core function affects the Csiszár f -divergence functional. A general treatment for the type of convexity has been considered and the associated perspective functions have been studied. In particular, it has been shown that when the core function is MN-convex, then the associated perspective function is jointly MN-convex if the two scalar means M and N are the same. In the case where $M \neq N$, we study the type of convexity of the perspective function. As an application, we prove that the *Hellinger distance* is jointly GG-convex.

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