

A SHARPENED VERSION OF ACZÉL INEQUALITY BY ABSTRACT CONVEXITY

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Abstract. In this study, the Aczél inequality is considered and a new simple proof of the inequality is provided. An extension and a sharper version of this inequality are obtained by performing the results based on the optimality conditions of abstract convex functions.

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