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# ON COMPLEX $L_p$ AFFINE ISOPERIMETRIC INEQUALITIES

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*Abstract.* Recently, Haberl [18] established the complex version of the Petty projection inequality and the Busemann-Petty centroid inequality. In this paper, we define the complex $L_p$ projection body operator $\Pi_{C,p}$ and the complex $L_p$ centroid body operator $\Gamma_{C,p}$. When $p \geqslant 1$ and $C$ is a complex $L_p$ zonoid in the complex plane, we establish the complex extension of the $L_p$ Busemann-Petty centroid inequality and the $L_p$ Petty projection inequality.

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