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THE UPPER BOUNDARY FOR THE RATIO BETWEEN n–VARIABLE OPERATOR POWER MEANS

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Abstract. In this paper, we show estimates of the upper boundary for the ratio between n-variable operator power means $P_t(\omega;\mathbb{A})$ due to Lawson-Lim-Pálfia by terms of a generalized condition number in the sense of Turing, which are partial improvements of the known results: Let $\mathbb{A} = (A_1,\dots,A_n)$ be a n-tuple of positive invertible operators such that $mI \leqslant A_j \leqslant MI$ for $j = 1,\dots,n$ and $h = M/m$, and ω a weight vector. Then

$$P_t(\omega;\mathbb{A}) \leqslant \left(\frac{h^t + h^{-t}}{2}\right)^{1/t} G_K(\omega;\mathbb{A})$$

for all $t \in (0,1]$, where $G_K(\omega;\mathbb{A})$ is the Karcher mean.

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