

WEIGHTED NORM INEQUALITIES FOR THE GENERALIZED MULTILINEAR STIELTJES TRANSFORMATION

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Abstract. We characterize some weighted strong and weak-type inequalities for the generalized Stieltjes and Calderón multilinear operators. As applications, we characterize a weighted multilinear Hilbert's inequality and a weighted Hilbert's multiple series theorem.

Mathematics subject classification (2020): 26D15.

Keywords and phrases: Calderón operator, Hilbert inequality, Hilbert operator, multilinear Hardy operators, multilinear Stieltjes transform, Stieltjes transform, weighted inequalities, weights.

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