

HERMITE–HADAMARD TYPE INEQUALITIES FOR MULTIDIMENSIONAL STRONGLY h -CONVEX FUNCTIONS

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Abstract. We establish some Hermite-Hadamard type inequalities for strongly h -convex function on balls and ellipsoids, which extend some known results. Some mappings connected with these inequalities and related applications are also obtained.

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