

## INHOMOGENEOUS LIPSCHITZ SPACES ASSOCIATED WITH FLAG SINGULAR INTEGRALS AND THEIR APPLICATIONS

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*Abstract.* This note is motivated by Müller, Ricci and Stein’s work in [29]. We introduce a new class of inhomogeneous Lipschitz spaces associated with flag singular integrals and characterize these spaces via the Littlewood-Paley theory. We prove that inhomogeneous flag singular integral operators are bounded on these Lipschitz spaces.

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