

## REFINEMENTS OF KY FAN'S EIGENVALUE INEQUALITY FOR SIMPLE EUCLIDEAN JORDAN ALGEBRAS BY USING GRADIENTS OF $K$ -INCREASING FUNCTIONS

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**Abstract.** In this paper, by using some analytical methods based on gradients of  $K$ -increasing functions, we show refinements of the following Ky Fan like inequality:  $\lambda(x+y) \prec \lambda(x) + \lambda(y)$  with elements  $x$  and  $y$  of a simple Euclidean Jordan algebra, the eigenvalue operator  $\lambda(\cdot)$  and Schur's majorization  $\prec$ .

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