

ON L_p INTERSECTION MEAN ELLIPSOIDS AND AFFINE ISOPERIMETRIC INEQUALITIES

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Abstract. In the paper, we discussed the L_p ($p \geq 1$) harmonic combination of convex bodies in \mathbb{R}^n . A variational formula for j th affine mean intersection $\bar{\Lambda}_j$ of convex bodies is established when $1 \leq j \leq n-1$. Using the new L_p intersection ellipsoids associated with convex bodies, some affine isoperimetric equalities are obtained.

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